Special Relativity

Module: The Nature of Light

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Overview

Einstein's special theory of relativity (1905) revolutionised our understanding of space, time, and the relationship between mass and energy. It is based on two simple postulates but leads to profound consequences.

Key Syllabus Points:

- Analyse Einstein's two postulates and their experimental evidence
- Investigate time dilation and length contraction with quantitative analysis
- Apply mass-energy equivalence $E = mc^2$

Einstein's Postulates

❗ The Two Postulates

1. **Principle of Relativity**: The laws of physics are the same in all inertial reference frames
2. **Constancy of Light Speed**: The speed of light in a vacuum ($c = 3 \times 10^8$ m/s) is the same for all observers, regardless of the motion of the source or observer

Experimental Evidence

Postulate	Evidence
Speed of light constant	Michelson-Morley experiment (1887)
Time dilation	Muon decay at Earth's surface
Time dilation	Hafele-Keating experiment (atomic clocks on planes)
Length contraction	Particle accelerator observations
Mass-energy	Nuclear reactions, particle physics

Key Concepts

The Lorentz Factor

The Lorentz factor γ appears in all relativistic equations:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Speed (v)	v/c	γ
0	0	1.00
0.5c	0.5	1.15
0.8c	0.8	1.67
0.9c	0.9	2.29
0.99c	0.99	7.09
0.999c	0.999	22.4

Time Dilation

> **ℹ Time Dilation**
>
> Moving clocks run slower. Time passes more slowly for a moving observer relative to a stationary observer.

$$t = \gamma t_0 = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where: - t = dilated time (measured by stationary observer) - t_0 = proper time (measured by observer at rest relative to events) - v = relative velocity

Proper time is always the shortest time interval, measured in the frame where the events occur at the same location.

Length Contraction

> **ℹ Length Contraction**
>
> Moving objects are shorter. Length contracts in the direction of motion.

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

Where: - l = contracted length (measured by stationary observer) - l_0 = proper length (measured in object's rest frame)

Proper length is always the longest length, measured in the frame where the object is at rest.

Relativistic Momentum

At high speeds, momentum increases without limit as $v \to c$:

$$p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This explains why particles cannot reach or exceed the speed of light.

Mass-Energy Equivalence

$$E = mc^2$$

For a particle at rest: $E_0 = m_0 c^2$ (rest energy)

Total relativistic energy: $E = \gamma m_0 c^2$

Kinetic energy at high speeds: $K = (\gamma - 1) m_0 c^2$

Worked Examples

Example 1: Muon Decay

Muons created in the upper atmosphere have a half-life of 2.2 s in their rest frame. They travel at 0.99c toward Earth. Calculate: (a) Their half-life as measured on Earth (b) How far they travel in one half-life (Earth frame)

Solution:

(a) Calculate γ:

$$\gamma = \frac{1}{\sqrt{1 - 0.99^2}} = 7.09$$

Dilated half-life:

$$t = \gamma t_0 = 7.09 \times 2.2 \times 10^{-6} = 15.6 \times 10^{-6} \text{ s}$$

(b) Distance in Earth frame:

$$d = vt = 0.99 \times 3 \times 10^8 \times 15.6 \times 10^{-6} = 4630 \text{ m}$$

Example 2: Length Contraction

A spaceship 100 m long (in its rest frame) travels past Earth at 0.8c. Calculate its length as measured from Earth.

Solution:

$$\gamma = \frac{1}{\sqrt{1-0.8^2}} = \frac{1}{\sqrt{0.36}} = 1.67$$

$$l = \frac{l_0}{\gamma} = \frac{100}{1.67} = 60 \text{ m}$$

Example 3: Mass-Energy

Calculate the energy released when 1.0 g of mass is converted to energy.

Solution:

$$E = mc^2 = (1.0 \times 10^{-3})(3 \times 10^8)^2$$
$$E = 9.0 \times 10^{13} \text{ J} = 90 \text{ TJ}$$

This is equivalent to about 21.5 kilotonnes of TNT!

Common Misconceptions

> ⚠ Avoid These Mistakes
>
> 1. **Mixing up frames** - Always identify which observer measures proper time/length
> 2. **Relativistic mass** - Modern physics uses rest mass m_0; "relativistic mass" is outdated
> 3. **Speed addition** - Velocities don't simply add at high speeds
> 4. **Both see other's clock slow** - This is NOT a contradiction; it depends on the reference frame
> 5. **Objects appear shorter** - Length contraction is a real effect, not an optical illusion

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HSC Exam Analysis

Question Types

1. **Calculation questions (5-7 marks)**: Calculate γ, dilated time, contracted length
2. **Evidence questions (4-5 marks)**: Describe experimental evidence for postulates
3. **Analysis questions (6-8 marks)**: Muon decay analysis, twin paradox explanation

Common Calculation Steps

1. Identify the proper quantity (measured in rest frame)
2. Calculate Lorentz factor γ
3. Apply appropriate equation
4. Check: dilated time $>$ proper time, contracted length $<$ proper length

Recent HSC Questions

- 2024 Q30: Muon decay analysis with both time dilation and length contraction
- 2023 Q29: Mass-energy calculation in particle collision
- 2022 Q28: Evidence for time dilation

Practice Problems

1. A particle travels at 0.6c. Calculate the Lorentz factor and the factor by which time is dilated.

2. Muons have a rest-frame half-life of 2.2 s. At what speed must they travel for their half-life to be 6.6 s as measured on Earth?

3. A meter stick travels at 0.9c. What length does a stationary observer measure?

4. Calculate the rest mass energy of a proton ($m_p = 1.67 \times 10^{-27}$ kg) in both joules and MeV.

5. A spaceship travels to a star 10 light-years away at 0.8c. Calculate the journey time as measured by (a) Earth observers and (b) spaceship passengers.

Related Topics

- The Photoelectric Effect
- Mass-Energy in Nuclear Reactions
- Evidence for Special Relativity