Projectile Motion

Module: Advanced Mechanics

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Overview

Projectile motion describes the motion of an object launched into the air that moves under the influence of gravity alone (ignoring air resistance). This is a key topic in Module 5 that builds on Year 11 kinematics.

Key Syllabus Points:

- Analyse the motion of projectiles by resolving motion into horizontal and vertical components
- Apply kinematic equations to projectile motion problems
- Investigate projectile motion experimentally

Key Concepts

Independence of Horizontal and Vertical Motion

> **!** Fundamental Principle
>
> The horizontal and vertical components of projectile motion are completely independent of each other. Gravity only affects the vertical component.

Component	Acceleration	Velocity
Horizontal	$a_x = 0$	$v_x = u_x = u\cos\theta$ (constant)
Vertical	$a_y = -g = -9.8$ m/s^2	$v_y = u_y + a_y t$ (changes)

Kinematic Equations

For horizontal motion (constant velocity):

$$x = u_x t = (u\cos\theta)t$$

For vertical motion (constant acceleration):

$$v_y = u_y + gt$$

$$y = u_y t + \frac{1}{2}gt^2$$

$$v_y^2 = u_y^2 + 2gy$$

Key Formulas Summary

Quantity	Formula
Initial horizontal velocity	$u_x = u\cos\theta$
Initial vertical velocity	$u_y = u\sin\theta$
Time of flight	$T = \frac{2u\sin\theta}{g}$
Maximum height	$H = \frac{u^2\sin^2\theta}{2g}$
Range	$R = \frac{u^2\sin(2\theta)}{g}$
Maximum range	At $\theta = 45°$

Worked Examples

Example 1: Ball thrown horizontally

A ball is thrown horizontally at 15 m/s from a cliff 80 m high. Find: (a) Time to hit the ground (b) Horizontal distance travelled (c) Velocity on impact

Solution:

(a) Using $y = u_y t + \frac{1}{2}gt^2$ with $u_y = 0$:

$$-80 = 0 + \frac{1}{2}(-9.8)t^2$$

$$t = 4.04 \text{ s}$$

(b) Horizontal distance:

$$x = u_x t = 15 \times 4.04 = 60.6 \text{ m}$$

(c) Final velocity components:

- $v_x = 15$ m/s (unchanged)
- $v_y = u_y + gt = 0 + (-9.8)(4.04) = -39.6$ m/s

Speed: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 39.6^2} = 42.3$ m/s

Angle: $\theta = \tan^{-1}\left(\frac{39.6}{15}\right) = 69.3°$ below horizontal

Example 2: Projectile launched at an angle

A projectile is launched at 30 m/s at 40° above horizontal. Calculate: (a) Maximum height (b) Time of flight (c) Range

Solution:

(a) Maximum height:

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{30^2 \sin^2(40°)}{2 \times 9.8} = 18.9 \text{ m}$$

(b) Time of flight:

$$T = \frac{2u \sin\theta}{g} = \frac{2 \times 30 \times \sin(40°)}{9.8} = 3.94 \text{ s}$$

(c) Range:

$$R = \frac{u^2 \sin(2\theta)}{g} = \frac{30^2 \sin(80°)}{9.8} = 90.4 \text{ m}$$

Common Misconceptions

⚠ Avoid These Mistakes

1. **Forgetting sign conventions** - Choose a consistent direction as positive (usually up) and stick with it
2. **Using $g = 10$ m/s² ** - In HSC, always use $g = 9.8$ m/s² unless told otherwise
3. **Confusing components** - Keep horizontal and vertical calculations separate
4. **Missing the projectile at different heights** - Formulas for T, H, R assume launch and landing at same height
5. **Air resistance** - In HSC Physics, assume negligible unless specifically stated

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HSC Exam Analysis

Question Types

1. **Calculation questions (4-6 marks)**: Calculate time, range, height, or velocity
2. **Graphing questions (3-4 marks)**: Sketch trajectory or velocity-time graphs
3. **Analysis questions (5-7 marks)**: Compare trajectories, explain independence principle

Recent HSC Questions

- 2024 Q21: Projectile calculation with non-level landing
- 2023 Q24: Projectile analysis comparing two trajectories
- 2022 Q23: Projectile launched from height

Practice Problems

1. A ball is kicked at 25 m/s at 60° above horizontal. Calculate the maximum height and range.
2. A projectile lands 100 m away after 4.0 s in the air. Calculate its initial speed and angle.
3. Compare the trajectories of two projectiles launched at 30° and 60° with the same speed.
4. A ball is thrown horizontally at 12 m/s from a 45 m cliff. How far from the base does it land?

Related Topics

- Circular Motion
- Gravitational Fields
- Motion in a Straight Line